EXERCISE – V

JEE PROBLEMS

1. Let $f(x) = \begin{cases} |x| & \text{for } 0 < |x| \le 2 \\ 1 & \text{for } x = 0 \end{cases}$. Then at x = 0, ' f ' has **[JEE 2000 (Scr.), 1]**

(A) a local maximum (B) no local maximum
(C) a local minimum (D) no extremum

2. Find the area of the right angled triangle of least area that can be drawn so as to circumscribe a rectangle of sides 'a' and 'b', the right angle of the triangle coinciding with one of the angles of the rectangle. **[REE 2001 Mains, 5]**

3. (a) Let $f(x) = (1 + b^2)x^2 + 2bx + 1$ and let m(b) is minimum value of f(x). As b varies, the range of m(b) is **[JEE 2001 (Scr.), 1 + 1]**

(A) [0, 1] (B) (0, 1/2] (C) [1/2,1] (D) (0, 1]

(b) The maximum value of $(\cos\alpha_1)\cdot(\cos\alpha_2)\ldots.(\cos\alpha_n)$, under the restrictions $0 \le \alpha_1, \alpha_2, \ldots\ldots\ldots, \alpha_n \le \frac{\pi}{2}$ and $\cot\alpha_1 \cdot \cot\alpha_2 \ldots\ldots\ldots\ldots \cot\alpha_n = 1$ is

(A) $\frac{1}{2^{n/2}}$ (B) $\frac{1}{2^n}$ (C) $\frac{1}{2n}$ (D) 1

4. If $a_1, a_2, \ldots\ldots, a_n$ are positive real numbers whose product is a fixed number e, the minimum value of $a_1 + a_2 + a_3 + \ldots\ldots + a_{n-1} + 2a_n$ is **[JEE 2002 (Scr.)]**

(A) $n(2e)^{1/n}$ (B) $(n + 1)e^{1/n}$
(C) $2ne^{1/n}$ (D) $(n + 1)(2e)^{1/n}$

5. (a) Find a point on the curve $x^2 + 2y^2 = 6$ whose distance from the line x + y = 7, is minimum. **[JEE 2003 Mains, 2 + 2]**

(b) For a circle $x^2 + y^2 = r^2$ find the value of 'r' for which the area enclosed by the tangents drawn from the point P(6, 8) to the circle and the chord of contact is maximum.

6. (a) Let $f(x) = x^3 + bx^2 + cx + d$, $0 < b^2 < c$. Then f **[JEE 2004, (Scr.)]**

(A) is bounded (B) has a local maxima
(C) has a local minima (D) is strictly increasing

(b) Prove that $\sin x + 2x \ge \frac{3x\cdot(x+1)}{\pi}\ \ \forall\ x \in \left[0, \frac{\pi}{2}\right]$.
(Justify the inequality, if any used). **[JEE 2004, 4]**

7. If p(x) be a polynomial of degree 3 satisfying p(−1) = 10, p(1) = −6 and p(x) has maximum at x = −1 and p′(x) has minima at x = 1. Find the distance between the local maximum and local minimum of the curve. **[JEE 2005 Mains, 4]**

8. (a) If f(x) is cubic polynomial which f(x) has local maximum at x = −1. If f(2) = 18 and f(1) = −1 and f ′(x) has local minima at x = 0, then **[JEE 2006, 5 + 5 + 6]**

(A) the distance between (−1, 2) and (a(f(a)), where x = a is the point of local minima is $2\sqrt{5}$
(B) f(x) is increasing for $x \in (1, 2\sqrt{5}]$
(C) f(x) has local minima at x = 1
(D) the value of f(0) = 5

(b) Let $f(x) = \begin{cases} e^x, & 0 \le x \le 1 \\ 2 - e^{x-1}, & 1 < x \le 2 \\ x - e, & 2 < x \le 3 \end{cases}$ and $g(x) = \int_0^x f(t)\,dt$, $x \in [1, 3]$ then g(x) has

(A) local maxima at x = 1 + ℓn 2 and local minima at x = e
(B) local maxima at x = 1 and local minima at x = 2
(C) no local maxima (D) no local minima

(c) If f(x) is twice differentiable function such that f(a) = 0, f(b) = 2, f(c) = −1, f(d) = 2, f(e) = 0, where a < b < c < d < e, then find the minimum number of zeros of $g(x) = (f'(x))^2 + f(x).f''(x)$ in the interval [a, e].

9. (a) The total number of local maxima and local minima of the function $f(x) = \begin{cases} (2+x)^3, & -3 < x \le -1 \\ x^{2/3}, & -1 < x < 2 \end{cases}$ is **[JEE 2008, 3 + 4 + 4 + 4]**

(A) 0 (B) 1 (C) 2 (D) 3

(b) Comprehension :
Consider the function $f : (-\infty, \infty) \to (-\infty, \infty)$ defined by

$$f(x) = \frac{x^2 - ax + 1}{x^2 + ax + 1}, \; 0 < a < 2$$

(i) Which of the following is true ?
(A) $(2 + a)^2 f''(1) + (2 - a)^2 f''(-1) = 0$
(B) $(2 - a)^2 f''(1) - (2 + a)^2 f''(-1) = 0$
(C) $f'(1) f'(-1) = (2 - a)^2$
(D) $f'(1) f'(-1) = -(2 + a)^2$

(ii) Which of the following is true ?
(A) f(x) is decreasing on $(-1, 1)$ and has a local minimum at $x = 1$
(B) f(x) is increasing on $(-1, 1)$ and has a local maximum at $x = 1$
(C) f(x) is increasing on $(-1, 1)$ but has neither a local maximum and nor a local minimum at $x = 1$.
(D) f(x) is decreasing on $(-1, 1)$ but has neither a local maximum and nor a local minimum at $x = 1$.

(iii) Let $g(x) = \int_0^{e^x} \frac{f'(t)}{1+t^2} dt$
Which of the following is true ?
(A) $g'(x)$ is positive on $(-\infty, 0)$ and negative on $(0, \infty)$
(B) $g'(x)$ is negative on $(-\infty, 0)$ and positive on $(0, \infty)$
(C) $g'(x)$ changes sign on both $(-\infty, 0)$ and $(0, \infty)$
(D) $g'(x)$ does not change sign on $(-\infty, \infty)$

10. (a) Let p(x) be a polynomial of degree 4 having extremum at x = 1, 2 and $\lim_{x\to 0}\left(1+\frac{p(x)}{x^2}\right) = 2$. Then the value of p(2) is **[JEE 2009, 4 + 4]**

(b) The maximum value of the function
$f(x) = 2x^3 - 15x^2 + 36x - 48$ on the set
$A = \{x \mid x^2 + 20 \le 9x\}$ is

11. (a) Let f, g and h be real-valued functions defined on the interval [0, 1] by $f(x) = e^{x^2} + e^{-x^2}$, $g(x) = xe^{x^2} + e^{-x^2}$ and $h(x) = x^2e^{x^2} + e^{-x^2}$. If a, b and c denote respectively, the absolute maximum of f, g and h on [0, 1], then **[JEE 2010, 3 + 3]**
(A) $a = b$ and $c \ne b$ (B) $a = c$ and $a \ne b$
(C) $a \ne b$ and $c \ne b$ (D) $a = b = c$

(b) Let f be a function defined on R (the set of all real numbers) such that
$f'(x) = 2010(x - 2009)(x - 2010)^2(x - 2011)^3(x - 2012)^4$, for all $x \in R$. If g is a function defined on R with values in the interval $(0, \infty)$ such that $f(x) = \ell n (g(x))$, for all $x \in R$, then the number of points in R at which g has a local maximum is

12. The number of distinct real roots of
$x^4 - 4x^3 + 12x^2 + x - 1 = 0$ is **[JEE 2011, 4]**

13. Let p(x) be a real polynomial of least degree which has a local maximum at x = 1 and a local minimum at x = 3. If p(1) = 6 and p(3) = 2, then p'(0) is **[JEE 2012]**

14. et $f : IR \to IR$ be defined as $f(x) = |x|+|x^2 - 1|$. The total number of points at which f attains either a local maximum or a local minimum is **[JEE 2012]**

15. If $f(x)=\int_0^x e^{t^2}(t-2)(t-3)\,dt$ for all $x \in (0, \infty)$, then
(A) f has a local maximum at x = 2 **[JEE 2012]**
(B) f is decreasing on (2, 3)
(C) there exists some $c \in (0, \infty)$ such that $f''(c) = 0$
(D) f has a local minimum at x = 3